**UNDERGRADUATE FOURTH SEMESTER (PROGRAMME) EXAMINATIONS, 2022**

**Subject: Mathematics** **Course ID: 42110**

**Course Code: SP/MTH/404/SEC-2** **Course Title: Graph Theory**

**Time: 2 hours** **Full Marks: 40**

**The figures in the margin indicate full marks**

**Notations and symbols have their usual meaning**

1.**Answer *any five* questions:** **2×5=10**

(a) Define the complement of a simple graph G with an example.

(b) Prove that the number of vertices of odd degree in a graph is always even.

(c) Find the adjacency matrix of complete bipartite graph $K_{3,3}$.

(d) Find the minimum and maximum number of edges of a simple graph with 10 vertices and 3 components.

(e) Define distance and centre in a tree with an example.

(f) Does there exist a tree of order $15$ such that the sum of the degrees of the vertices is $30$? Justify your answer.

(g) Does there exist a simple graph with 5 vertices having degrees $2, 2, 4, 4, 4$? Justify your answer.

(h) Give an example of a connected Eulerian graph which is not Hamiltonian.

2. **Answer *any four* questions:** **5×4=20**

(a) (i) Define connected graph.

(ii) Prove that a simple graph with $n$ vertices and $k$ components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

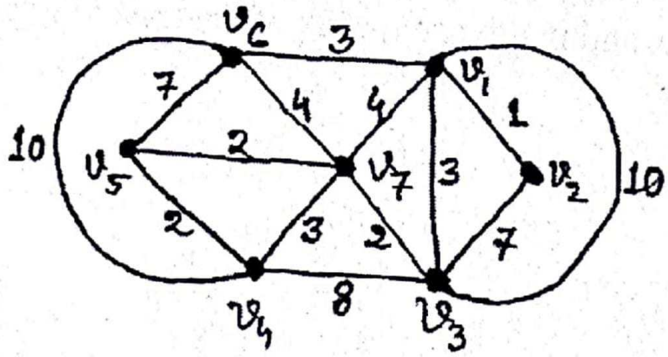
1+4

(b) (i) Prove that a graph $G$ contains a circuit, if the degree of each vertex of $G$ is at least two.

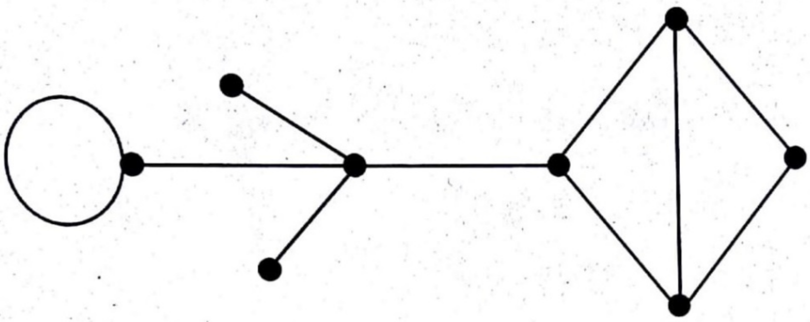
(ii) Find the maximum number of vertices in a simple graph with $35$ edges and degree of each vertex is at least $3$. 3+2

(c) Define the adjacency matrix and incidence matrix of a graph. With proper example, show that the adjacency matrix of a graph and incidence matrix of a graph may be same. 2+3

(d) Apply Dijkstra's algorithm to find the shortest path(with length) from $v_2$ to $v_5$of the following graph:




(e) Find all the spanning trees of the following graph:



(f) (i) Define a minimally connected graph.

(ii) Prove that a graph is minimally connected if and only if it is a tree. 1+4

3. **Answer *any one* question:** $\mathbf{10 \times 1 = 10}$

(a) (i) If a graph (connected or disconnected) has exactly two vertices of odd degree , then prove that there must be a path joining these two vertices .

(ii) Prove that for an $n$ −vertex graph $G$ (with $n \geq 1$) , the following are equivalent:

(I) $G$ is connected having no cycles.

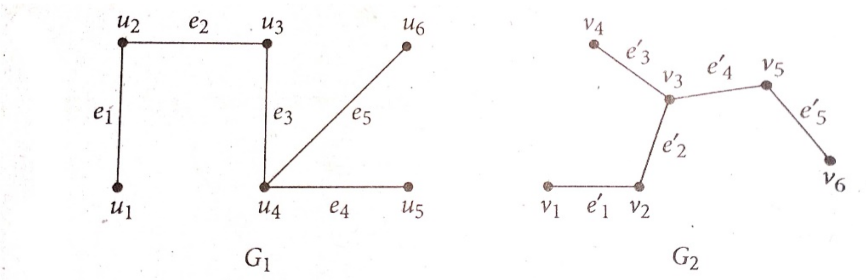
(II) $G$ is connected having $n - 1$ edges.

(III) $G$ has $n - 1$ edges with no cycle. 4+6

(b) (i) Show that in a directed graph sum of the in-degrees and sum of the out-degrees of the vertices are same.

(ii) Show that a graph is connected if and only if $G$ has a spanning tree.

(iii) Examine whether the following two graphs $G_1$ and $G_2$ are isomorphic:




3+4+3

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